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Mark Scheme (Results)
January 2010

GCE

Further Pure Mathematics FP1 (6674)

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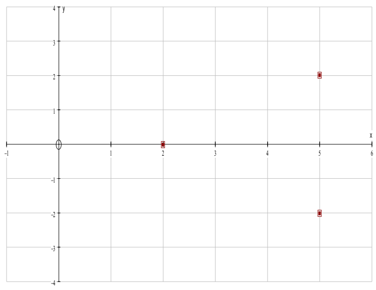
Question Number	Scheme	Marks
Q1	(a) $\frac{a+\mathrm{i}b}{3-\mathrm{i}} \times \frac{3+\mathrm{i}}{3+\mathrm{i}} = \frac{(3a-b)+\mathrm{i}\,(a+3b)}{10}$ (A1 for numerator, A1 for 10)	M1 A1 A1 (3)
	(b) $\lvert z_1 \rvert = \sqrt{a^2+(-2a)^2} = \sqrt{5a^2} = a\sqrt{5}$ (*)	M1 A1 (2)
	(c) $\arg\frac{z_1}{z_2} = \arctan\frac{a+3b}{3a-b} = \arctan(\text{-}1), \quad = -\frac{\pi}{4} \quad \left(\text{or } \frac{7\pi}{4}, \text{ or } -45°, \text{ or } 315°\right)$	M1 A1ft, A1 (3) [8]
	(c) The <u>final</u> A1 requires a single answer, so for example: $\arctan(\text{-}1) = -\frac{\pi}{4}$ or $\frac{3\pi}{4}$ is A0	

Question Number	Scheme	Marks
Q2	(a) $f(2) = 2\cos 2 - 4 + 5 \qquad (= 0.1677...)$	
	$f(2.1) = 2.1\cos 2.1 - 4.2 + 5 \quad (= -0.2601...)$	M1
	Values correct (to 1 s.f.). Change of sign $\Rightarrow$ Root	A1 (2)
	(b) $f'(x) = \cos x - x\sin x - 2$	M1 A1
	$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 2 - \dfrac{0.1677}{-4.2347}, \qquad = 2.04$	M1 A1, A1 (5)
	(c) $f(2.035) = ...$ and $f(2.045) = ...$	M1
	$0.0189...$ and $-0.0238...$ Change of sign $\Rightarrow$ Correct to 2 d.p.	A1 (2) [9]
	(c) The M1 is also given for evaluating f at the ends of a 'tighter' interval.	

Question Number	Scheme	Marks
Q3	(a) $5-2\mathrm{i}$ is a root	B1 (1)
	(b) $\left(x-(5+2\mathrm{i})\right)\left(x-(5-2\mathrm{i})\right)=x^2-10x+29$	M1 M1
	$x^3-12x^2+cx+d=\left(x^2-10x+29\right)\left(x-2\right)$	M1
	$c=49,\qquad d=-58$	A1, A1 (5)
	(c)  Conjugate pair in 1st and 4th quadrants, (symmetrical about real axis).	B1
	Fully correct, labelled.	B1 (2)
		[8]
	(b) 1st M: Form brackets using $(x-\alpha)(x-\beta)$ and expand. 2nd M: Achieve a 3-term quadratic with no i's.	
	(b) Alternative: Substitute a root (usually $5+2\mathrm{i}$) and expand brackets $(5+2\mathrm{i})^3-12(5+2\mathrm{i})^2+c(5+2\mathrm{i})+d=0$	M1
	$(125+150\mathrm{i}-60-8\mathrm{i})-12(25+20\mathrm{i}-4)+(5c+2c\mathrm{i})+d=0$ (2nd M for achieving an expression with no powers of i)	M1
	Equate real and imaginary parts	M1
	$c=49,\qquad d=-58$	A1, A1

Question Number	Scheme	Marks
Q4	$m^2 + 6m + 9 = 0 \qquad m = -3$	B1
	C.F. $(x =)\ (At + B)e^{-3t}$	M1 A1
	P.I. $x = p\cos t + q\sin t$	B1
	$\frac{dx}{dt} = -p\sin t + q\cos t \qquad \frac{d^2x}{dt^2} = -p\cos t - q\sin t$	M1
	$-p\cos t - q\sin t - 6p\sin t + 6q\cos t + 9p\cos t + 9q\sin t = 5\cos t$	M1
	$-6p + 8q = 0$ and $8p + 6q = 5$	A1
	Solve simultaneously to find either p or q:	M1
	$p = \frac{2}{5}$ and $q = \frac{3}{10}$	A1
	General solution: $(x =) \quad (At + B)e^{-3t} + \frac{2}{5}\cos t + \frac{3}{10}\sin t$	A1ft
		[10]
	The final A1ft is dependent on the 3 preceding M marks (for the P.I.)	

Question Number	Scheme	Marks
Q5	(a) $1 = A(2r+5) + B(2r+1)$, and find values of A and B	M1
	$\frac{1}{4(2r+1)} - \frac{1}{4(2r+5)}$	A1 (2)
	(b) $r=1$: $\frac{1}{4}\left(\frac{1}{3}-\frac{1}{7}\right)$	B1ft
	$r=2$: $\frac{1}{4}\left(\frac{1}{5}-\frac{1}{9}\right)$	
	$r=n-1$: $\frac{1}{4}\left(\frac{1}{2n-1}-\frac{1}{2n+3}\right)$	
	$r=n$: $\frac{1}{4}\left(\frac{1}{2n+1}-\frac{1}{2n+5}\right)$	
	Sum: $\frac{1}{4}\left(\frac{1}{3}+\frac{1}{5}-\frac{1}{2n+3}-\frac{1}{2n+5}\right)$	M1 A1
	$=\frac{1}{4}\left(\frac{8(2n+3)(2n+5)-15(2n+5)-15(2n+3)}{15(2n+3)(2n+5)}\right)$	M1
	$=\frac{1}{4}\left(\frac{32n^2+128n+120-30n-75-30n-45)}{15(2n+3)(2n+5)}\right)$	A1
	$=\frac{1}{4}\left(\frac{32n^2+68n}{15(2n+3)(2n+5)}\right)$	
	$=\frac{n(8n+17)}{15(2n+3)(2n+5)}$ $(c = 17)$	A1 (6)
		[8]
	(b) B1ft for one correct difference (ft A and B). M1 A1 M1 A1: The $\frac{1}{4}$ is not needed for these marks, only for the final A1.	

Question Number	Scheme	Marks
Q6	(a) $r=\frac{a}{2}$ $\quad \sin 2\theta=\frac{1}{2}$ $\quad 2\theta=\frac{\pi}{6}$ or $\frac{5\pi}{6}$ $\quad \theta=\frac{\pi}{12},\ \frac{5\pi}{12}$	M1 A1, A1 (3)
	(b) $\sin^2 2\theta=\frac{1}{2}(1-\cos 4\theta)$	B1
	$\frac{a^2}{2}\int \sin^2 2\theta \,\mathrm{d}\theta=$	M1
	$\pm\left[\theta-\frac{\sin 4\theta}{4}\right]$ (Correct integration of $\pm(1-\cos 4\theta)$)	A1
	$\left[\ ....\ \right]_{\pi/12}^{5\pi/12}=.......,=\frac{a^2}{4}\left(\frac{5\pi}{12}-\left(-\frac{\sqrt{3}}{8}\right)-\frac{\pi}{12}+\frac{\sqrt{3}}{8}\right)=a^2\left(\frac{\pi}{12},+\frac{\sqrt{3}}{16}\right)$	M1, A1, A1 (6)
		[9]
	(b) 1st M: Use of $\frac{1}{2}\int r^2\,\mathrm{d}\theta$ with some integration attempt. 2nd M: Correct use of their limits. N.B. Other methods are possible, e.g. $\left(\text{e.g. } 2\left[\ ....\ \right]_{"\pi/12"}^{\pi/4}\right)$ Slips such as omitting the a or not squaring the a: just the final A mark is lost.	

Question Number	Scheme	Marks
Q7	(a) $10+3x-x^2=3x-1$ $\quad x^2=\ldots$ or $x=\ldots,$ $\quad \sqrt{11}\ (\beta)$	M1, A1
	$10+3x-x^2=1-3x$	M1
	$x^2-6x-9=0$ $\quad x=\dfrac{6\pm\sqrt{72}}{2}$ or equiv.	A1
	$3-3\sqrt{2}$ or exact equiv. (α)	A1 (5)
	(b) $3-3\sqrt{2}<x<\sqrt{11}$	M1 A1ft (2)
	(c) Forming inequalities using all their four x values $\quad \left(\pm\sqrt{11}\ \text{and}\ 3\pm3\sqrt{2}\right)$	M1
	$-\sqrt{11}<x<3-3\sqrt{2}$, $\quad \sqrt{11}<x<3+3\sqrt{2}$	B1, B1 (3) [10]
	Answers with decimals (3 s.f. accuracy) are acceptable in (b) and (c). (b) M: Answer including $x<\beta$ (positive β) or $x>\alpha$ (negative α). A1ft requires negative α and positive β.	

Question Number	Scheme	Marks
Q8	(a) $z = \frac{1}{y^2}$ $\quad \frac{dz}{dx} = -\frac{2}{y^3}\frac{dy}{dx}$	B1
	$-\frac{y^3}{2}\frac{dz}{dx} + y = 4xy^3$	M1
	$-\frac{y^2}{2}\frac{dz}{dx} + 1 = 4xy^2$ $\quad -\frac{1}{2z}\frac{dz}{dx} + 1 = \frac{4x}{z}$, $\quad \frac{dz}{dx} - 2z = -8x$ (*)	M1, A1 (4)
	(b) Integrating factor $e^{\int -2dx} = e^{-2x}$	B1
	$ze^{-2x} = -8\int xe^{-2x}\,dx$ or $\frac{d}{dx}\left(ze^{-2x}\right) = -8xe^{-2x}$	M1
	$\int xe^{-2x}\,dx = \left\{\frac{xe^{-2x}}{-2} + \frac{1}{2}\int e^{-2x}\,dx\right\}$	M1 A1
	$ze^{-2x} = 4xe^{-2x} + 2e^{-2x} + C$, $\quad z = 4x + 2 + Ce^{2x}$	M1, dM1
	(The second of these M marks is dependent on the first, and both are dependent on the use of an integrating factor).	
	$y = \frac{1}{\sqrt{4x + 2 + Ce^{2x}}}$ (or equiv.)	A1 (7)
	(c) $\frac{dy}{dx} = 0$: $\quad y = 4xy^3$ $\quad y = \frac{1}{2\sqrt{x}}$ (*)	M1 A1 (2)
		[13]
	(b) Alternative for first 6 marks: C.F. $z = Ce^{2x}$ — B1 P.I. $z = px + q$, $\quad \frac{dz}{dx} = p$ — M1 $p - 2px - 2q = -8x$ — M1 $p = 4 \quad q = 2$ — M1 A1 $z = 4x + 2 + Ce^{2x}$ — M1	

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